Identity with sum of binomial coefficients.

https://www.linkedin.com/feed/update/urn:li:activity:6741625579570827264

Show that

$$\binom{n+1}{k+1} = \binom{n}{k} + \binom{n-1}{k} + \ldots + \binom{k}{k}.$$

Solution by Arkady Alt, San Jose, California, USA.

Since $\binom{n}{m} = \binom{n-1}{m} + \binom{n-1}{m-1}, \forall n, m \in \mathbb{N}$ then

$$\sum_{i=k+1}^{n} \binom{i}{k} = \sum_{i=k}^{n} \left(\binom{i+1}{k+1} - \binom{i}{k+1} \right) = \binom{n+1}{k+1} - \binom{k+1}{k+1} = \binom{n+1}{k+1} - \binom{k}{k} \Leftrightarrow$$

$$\binom{n+1}{k+1} = \sum_{i=k}^{n} \binom{i}{k}.$$